

B.E.2nd Semester Examination, December-2008
Paper : PHY-102-E

Time allowed : 3 hours]

[Maximum marks : 100

Note:- Attempt any **five** questions in all, taking at least **two** from each part.

PART-A

1. (a) Write note on Laue method for crystal structure determination. 10
(b) Derive expression for Frenkel's defect in solid. 10
2. (a) Show that group velocity $V_g = -\lambda^2 \frac{dv}{d\lambda}$, where the symbols have their usual meanings. 5
(b) Derive the expression for Schrodinger time independent and time dependent wave equations. 15
3. (a) What are limitations of classical free electron theory of metal? 5
(b) Discuss the quantum theory of free electrons in one dimensional potential box. 15
4. (a) Write short note on quarks and gluons. 10
(b) Discuss X-ray diffraction powder method for crystal structure analysis. 10

PART - B

5. (a) What are E-K diagrams? What do you mean by -ve mass? Explain. 8
(b) What is Hall effect? Give an elementary theory of Hall effect. Also prove that $\tan \theta_H = \frac{R_H}{\mu}$ where θ_H is Hall angle, μ is mobility of charge carrier and H the applied magnetic field. 12
6. (a) Explain Langevin theory of paramagnetism and also discuss Weiss modification in Langevin theory of paramagnetism. 15
(b) What do you mean by magnetic susceptibility? Write down Curie-Weiss law. 5
7. (a) Derive the London equation and discuss how its solution explains Meissner effect? 15
(b) Discuss some applications of super conductors. 5
8. Write short notes on :
 - (a) Weiss theory of Ferromagnetism
 - (b) Effect of illumination on photo conductivity with and without concept of traps. 10

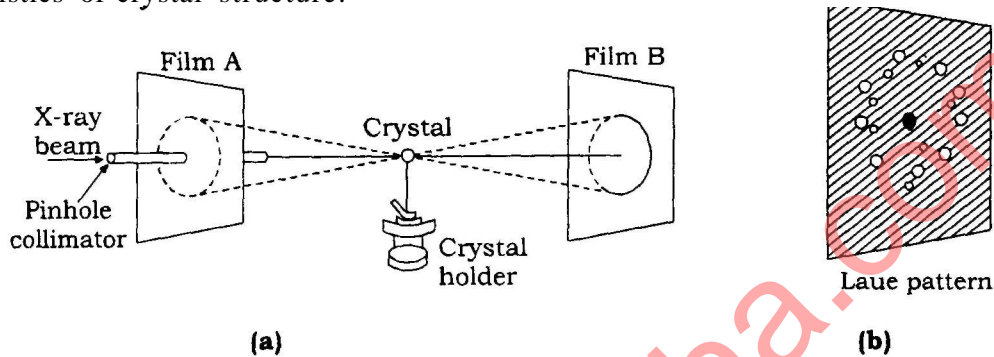
ANSWERS

Axis. 1. (a) Laue Method for Crystal Structure Determination :

Experimental Arrangement : It consists of

- (i) A continuous X-rays beam, well-collimated by a pin-hole.
- (ii) A single crystal held stationary at proper orientation relative to incident X-ray beam.
- (hi) A flat film to receive the transmitted (or reflected) diffraction beam.

Working Principle : When X-ray beam falls on the crystal, the beam is diffracted. The transmitted beam forms a series of spots on the film called Laue spots which are characteristics of crystal structure.



Each spot in the Laue diffraction pattern corresponds to an interference maxima for set of crystal planes satisfying the Bragg equation $2d \sin \theta = n\lambda$ for a particular wavelength selected from the continuous incident beam. From the position and intensities of these spots, one can determine crystal structure *i.e.*, the values of primitives a , b , c and the details of unit cell of the solid.

Ans. 1. (b) Expression for Frenkel Defects in solids

Frenkel defects are dominant in open structures such as silver halides. Consider a perfect crystal composed of equal number of positively and negatively charged ions. Let N be the total number of atoms, N_i the number of interstitial sites and E_i the energy required to displace a cation from its regular position to an interstitial position.

The total number of ways in which n Frenkel defects can be formed will be given by

$$W = \left\{ \frac{N!}{(N-n)!n!} \right\} \times \frac{N_i!}{(N_i-n)!n!} \quad \dots(1)$$

The corresponding increase in entropy, from Boltzmann relation is given by

$$S = k_B \log W = k_B \log \left[\left\{ \frac{N!}{(N-n)!n!} \right\} \times \left\{ \frac{N_i!}{(N_i-n)!n!} \right\} \right] \quad \dots(2)$$

$$\text{Internal energy, } U = n E_i \quad \dots(3)$$

The Change in free energy $F = U - TS$

$$nE_i - T k_B \log \left[\left\{ \frac{N!}{(N-n)!n!} \right\} \times \left\{ \frac{N_i!}{(N_i-n)!n!} \right\} \right] \quad \dots(4)$$

Now we simplify logarithmic term

$$\log \left\{ \frac{N!}{(N-n)!n!} \right\} \times \left\{ \frac{N_i!}{(N_i-n)!n!} \right\} = \log \left\{ \frac{N!}{(N-n)!n!} \right\} + \log \left\{ \frac{N_i!}{(N_i-n)!n!} \right\}$$

$$= \log N! - \log(N-n)! - \log n! + \log N_i! - \log(N_i-n)! - \log n!$$

Using Stirling's approximations for large numbers, i.e. $\log n! = n \log n - n$

We get ,

$$\begin{aligned} & \log \left\{ \frac{N!}{(N-n)!n!} \right\} \times \left\{ \frac{N_i!}{(N_i-n)!n!} \right\} \\ &= N \log N - N - (N-n) \log(N-n) + N - n - n \log n + n \\ &+ N_i \log N_i - N_i - (N_i-n) \log(N_i-n) + (N_i-n) - n \log n + n \\ &= N \log N + N_i \log N_i - (N-n) \log(N-n) - (N_i-n) \log(N_i-n) - 2n \log n \end{aligned}$$

∴ In view of this, equation (4) for free energy gives

$$F = nE_i - k_B T [N \log N + N_i \log N_i - (N-n) \log(N-n) - (N_i-n) \log(N_i-n) - 2n \log n] \quad \dots(5)$$

In equilibrium the free energy is constant,

$$\therefore \frac{\partial F}{\partial n} = 0$$

Differentiating (5) w.r.t. n and putting equal to 0, i.e.

$$\begin{aligned} E_i - k_B T \times [-(N-n) \times \frac{1}{(N-n)} \times (-1) - \log(N-n) \times (-1) - (N_i-n) \times \frac{1}{(N_i-n)} \times (-1) \\ - \log(N_i-n) \times (-1) - 2n \times \frac{1}{n} - 2 \log n] = 0 \end{aligned}$$

$$\Rightarrow E_i - k_B T [\log(N-n) + \log(N_i-n) - 2 \log n] = 0$$

$$\Rightarrow E_i - k_B T \left[\frac{\log(N-n)(N_i-n)}{n^2} \right] = 0$$

$$\Rightarrow E_i = k_B T \left[\log \frac{(N-n)(N_i-n)}{n^2} \right] \quad \dots(6)$$

Taking $n \ll N$ and $n \ll N_i$, we get

$$\begin{aligned} E_i &= k_B T \log \frac{NN_i}{n^2} \\ \Rightarrow E_i &= k_B T [\log NN_i - 2 \log n] \end{aligned}$$

$$\Rightarrow \log n = \frac{1}{2} \log NN_i - \frac{E_i}{2k_B T}$$

$$\Rightarrow \log \frac{N}{(NN_i)^{1/2}} = -\frac{E_i}{2k_B T}$$

$$\Rightarrow \frac{n}{(NN_i)^{1/2}} = e^{-E_i/2k_B T}$$

$$n = (NN_i)^{1/2} e^{-E_i/2k_B T}$$

This is the expression for the equilibrium concentration of Frenkel's defects at temperature T .

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Ans. 2. (a) The group velocity $v_g = \frac{d\omega}{dk}$

and Phase velocity $u = \frac{\omega}{k}$

To obtain a relation between group velocity v_g and wave velocity u , we put $k = \frac{2\pi}{\lambda}$ so that

$$v_g = \frac{d\omega}{d\left(\frac{2\pi}{\lambda}\right)} = -\frac{\lambda^2}{2\pi} \frac{d\omega}{d\lambda} \quad \dots(1)$$

Putting $\omega = 2\pi\nu$, we get

$$v_g = -\lambda^2 \frac{d\nu}{d\lambda} \quad \dots(2)$$

Ans. 2. (b) Time Independent Schrodinger Equation-

Consider a system of stationary waves to be associated with the particle. Let $\psi(r, t)$ be the wave characteristic for the de-Broglie wave at the any location $r = ix + jy + kz$ at time t . Then the differential equation of the wave motion in three dimension in accordance with Maxwell wave-equation can be written as

$$\nabla^2 \psi = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial x^2} \quad \text{or} \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad \dots(1)$$

where u is the wave velocity.

The solution of equation (1) gives ψ as a periodic displacement in term of time i.e.

$$\psi(r, t) = \psi_0 e^{-i\omega t} \quad \dots(2)$$

where ψ_0 is the amplitude at the point considered. It is function of position r i.e. of co-ordinates (x, y, z) and not of time t

The equation (2) may be expressed as

$$\psi(r, t) = \psi_0(r) e^{-i\omega t} \quad \dots(3)$$

Differentiating equation (3) twice with respect to t , we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0(r) e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0(r) e^{-i\omega t}$$

Substitute this in equation (1), we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{\omega^2}{u^2} \psi \quad \dots(4)$$

$$\omega = 2\pi\nu = \frac{2\pi u}{\lambda} \quad \Rightarrow \quad \frac{\omega}{u} = \frac{2\pi}{\lambda} \quad \dots(5)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \nabla^2 \psi \quad \dots(6)$$

using equations (5) and (6), equation (4) becomes

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$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \dots(7)$$

So far we have introduced wave mechanical concept and so that treatment is general. For introducing the concept of wave mechanics, we must put for De-Broglie equation.

$$\lambda = \frac{h}{mv} \quad \dots(8)$$

Sub. in equation (7), we get

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \quad \dots(9)$$

If E & V are the total and potential energies of the particle respectively then its K.E. $\frac{1}{2}mv^2$ is given by $\frac{1}{2}mv^2 = E - V$

which gives $m^2 v^2 = 2m(E - V)$

Sub. this in equation (9), we get

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \quad \dots(10)$$

The above equation is called Schrodinger time independent wave equation. The quantity ψ is usually referred as wave function.

Let us now substitute in eq. (10)

$$h = \frac{h}{2\pi} \quad \dots(11)$$

Then Schrodinger time-independent wave equation in usually used form may be written as

$$\nabla^2 \psi + \frac{2m}{h^2} (E - V) \psi = 0 \quad \dots(12)$$

This is Schrodinger's time independent wave equation.

Time-dependent Schrodinger Equation : Time-dependent Schrodinger equation may be obtained by eliminating E from equation (12).

Differentiating equation (3) with respect to t , we get

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= -i\omega \psi_0(\mathbf{r}) e^{-i\omega t} \\ &= -i(2\pi\nu) \psi_0(\mathbf{r}) e^{-i\omega t} && \text{(since } \omega = 2\pi\nu) \\ &= -2\pi i\nu \omega(\mathbf{r}) && \text{[Using (3)]} \\ &= -\frac{2\pi i E}{h} \psi && \left(\text{since } E = h\nu, \text{ i.e., } \nu = \frac{E}{h} \right) \\ &= -\frac{iE}{h} \psi \frac{i}{i} && \text{[Using (11)]} \end{aligned}$$

which gives $E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(13)$

Substituting value of $E\psi$ from above equation in (12), we get

$$\nabla^2\psi + \frac{2m}{\hbar^2} \left[i\hbar \frac{\partial\psi}{\partial t} - V\psi \right] = 0$$

or
$$\nabla^2\psi = -\frac{2m}{\hbar^2} \left[i\hbar \frac{\partial\psi}{\partial t} - V\psi \right]$$

i.e.
$$-\frac{\hbar^2}{2m} \nabla^2\psi + V\psi = i\hbar \frac{\partial\psi}{\partial t} \quad \dots(14)$$

This equation may be written as

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi = i\hbar \frac{\partial\psi}{\partial t} \quad \dots(15)$$

This equation contains the time and hence is called **time dependent Schrödinger Equation**.

Ans. 3. (a) Limitations of Free Electron Theory :

The free electron theory has the following limitations :

(i) **Specific heat.** The electronic contribution of specific heat has serious disagreement with experimental results.

The free electron theory could not explain why the low temperature specific heat should be dominated by the electronic contribution. However it is evident from experiment that T^3 correction to linear term is very definitely dominated by something else, since the simple Sommerfeld theory for the electronic contribution to the T^3 -term has the wrong sign and is millions of times too small.

(ii) **Paramagnetism of Metals :** Experimental fact that the paramagnetism of metals is nearly independent of temperature could not be explained by free electron theory.

(iii) **Wiedemann-Franz law :** The Wiedemann-Franz law, consequence of free electron theory, is obeyed beautifully at high (room) temperature and also quite probably at very low temperatures. At intermediate temperatures it fails and $\frac{K}{\sigma T}$ depends on temperature.

(iv) **Hall coefficient :** Free electron theory predicts a Hall coefficient which at metallic density of electrons has a constant value $R_H = -\frac{1}{ne}$, independent of temperature, relaxation time or the strength of the magnetic field. Although observed hall coefficient has this order of magnitude, but they depend upon high magnetic field strength and the temperature (and hence on relaxation time).

(v) **The magnetic resistance :** Free electron theory predicts that the resistance of a wire perpendicular to a uniform magnetic field should not depend upon the strength of the field; but in almost all cases it does. In some cases (e.g. noble metals : copper, silver and gold) it can be made to increase apparently without limit as the field increases.

(vi) **Temperature dependence of electrical conductivity :** Free electron theory does not account the temperature dependence of electrical conductivity.

(vii) **Directional dependence of electrical conductivity :** In some metals the d.c. electrical conductivity depends on the orientation of the specimen (if suitably prepared) with respect to electrical field. In such specimens the current density J need not even be parallel to the electric field E .

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Ans. 3. (b) Quantum theory of free electrons in one dimensional potential box :

Then assumptions of the theory are

1. The metal contains a large number of conduction electrons which are not completely free but partially so, in the sense that though they are not bound to any particular atomic system, yet are bound to the metal as a whole. The interior of the metal is to be conceived as a region of uniform potential with higher potential at the surface much like the molecules of an ideal gas.

The potential function, according to this mode, is shown in fig.

E_s is the energy difference between an electron at rest inside metal and in vacuum.

At absolute zero, all energy levels upto E_f are filled and all higher ones are empty.

The work function, i.e., work required to be done to extract an electron from metal,

$$W = E_s - E_f.$$

2. The forces between conduction electrons and ion cores are neglected in the free-electron approximation, so that the electron within the metal is treated as free. The total energy of electron is wholly kinetic energy since potential energy is negligible.

3. Due to their light mass and dense packing the electrons in metals may be considered as a gas under very high compression and hence to a degenerate gas. Moreover as the electron-gas is charged, the free-electron gas in metal may be regarded as *dense plasma*.

4. The electrons are assumed to obey Pauli's exclusion principle and hence they obey *Fermi-Dirac statistics* rather than the classical Boltzmann statistics. The most important characteristic of Fermi-Dirac statistics are the slightness of the dependence of distribution on temperature and the occurrence of zero point energy. The latter property is closely connected with Pauli principle.

Assuming free electron within a metal ($V_0 = 0$) and using solution of Schrödinger equation for this case, we get the allowed energy values of electrons within metal are

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

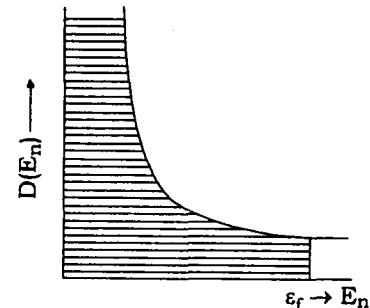
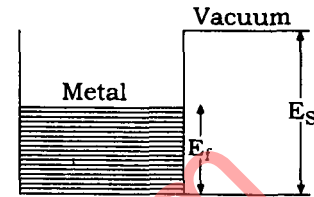
and the eigen-function is

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

As electron can occupy two spin states, there are two quantum states for a single electron, therefore density of states for this case would be given by

$$D(E_n) = \frac{dn}{dE_n} = 2 \cdot \frac{L}{\pi \hbar} \left(\frac{m}{2E_n} \right)^{1/2}$$

This result is plotted in fig.



Ans. 4. (a) Quarks : In 1964 Gellman and G. Ziverv independently proposed that at strongly interacting particles are built up of three new hitherto undiscovered particles called the **quarks**. The quarks are assumed to have fractional charges, fractional baryon numbers and

spin 1/2 each. On account of spin 1/2, they obey Fermi-Dirac statistics and so they are fermions. All baryons, antibaryons are composed of three kinds of quarks/antiquarks and all mesons, are formed of a quark and an antiquark of different kinds.

Originally three quarks were proposed.

a up quark (u), a downquark (d) and a strange quark (s). Then three new quarks were added to quark-family. They are charm quark (c), beauty (or bottom) quark (b) and truth (or top) quark (t). The top quark (t) was first identified at Femi-lab in 1994 and confirmed in 1995.

It was assumed that a proton is a combination of three quarks uud , while a neutron is made of udd . a π meson is a bound state of u and d while π^0 state is a bound state of d and \bar{u} . (\bar{u} is antiquark of quark u .)

Properties of quarks:

Name	Charge	Isospin		Baryon number B	Strangeness S	Charm	Bottom	Top
		T	T _z					
d	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
u	$+\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	0	0	0
s	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	-1	0	0	0
c	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0	1	0	0
b	$-\frac{1}{3}$	0	0	$\frac{1}{3}$	0	0	-1	0
t	$\frac{2}{3}$	0	0	$\frac{1}{3}$	0	0	1	1

Gluons : In 1973 H.D. Politzer and F. Wilczek, while studying the properties of Yang-mills theories of interactions among quarks made a remarkable discovery that in many circumstances, the effective strength of interaction becomes weaker at short distances. This implies that the interaction between the quarks becomes weaker at smaller separations.

According to this theory the quarks of a given colour create a field (called the gluon field) around them so that they emit or reabsorb a type of hypothetical particles, much like the emission and reabsorption of virtual photons by electrically charged particles. These hypothetical particles are called gluons.

Properties of gluons :

- (i) They have zero isospin ($I = 0$)
- (ii) The other properties of gluons resemble with that of photons, that is charge $Q = 0$, magnetic moment $m = 0$, charge parity $C = -1$.
- (iii) Unlike electrically neutral particles, the gluons carry a type of charge called the colour.

Ans. 4. (b) X-ray Diffraction Powder Method for Crystal Structure Analysis :

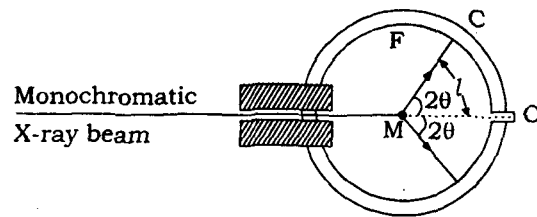
This method is used in crystal structure analysis to determine the various d -values for the crystals of a given material.

A narrow beam of monochromatic X-ray enters a cylindrical chamber C through a pin hole (not slit) P and falls on the powder sample (i.e., a collection of tiny crystals, compressed in the form of a cylindrical rod) which is set along the axis of the cylindrical chamber. The photographic film-strip F is put around the inner circumference of the chamber.

In the powder the million of micro-crystals have all possible orientations in space.

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When the collimated X-ray beam falls upon the powder, for each set of d and n , there is a particular value of θ , which satisfies Bragg's equation. For this value of θ , the beam appears at the corresponding 2θ deviation. As the parallel lattice planes with a given spacing d and same value of n and θ occur in all positions around the axis of the incident beam, the reflected rays produce a cone with a semi-vertical angle 2θ . For various sets of d and n , we obtain various cones of rays. One such cone with a semi-vertical angle 2θ is shown in the fig. which shows pattern recorded on the photographic film.



If l is the distance between the arcs of the same circle on the photographic film and R is the radius of the camera chamber, we have



$$2\theta_{rad} = \frac{1}{R} \quad \text{or} \quad \theta_{rad} = \frac{1}{2R}$$

Thus by measuring the diameters of these arcs ($2l$) and using the radius R of the camera chamber, we can determine θ .

Thus using the known values of θ , calculated values of d and properly selected value of n , the lattice spacing d can be deduced for a given crystal. For accurate determination of d , n must be large. Moreover if l or θ is large, the determination of d also accurate because for such values, d varies rapidly with changes of θ .

PART - B

Ans. 5. (a) E-K diagrams :

The condition for existence of solution of Schrodinger equation for periodic potential is

given by
$$P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos k a \quad \dots(1)$$

where
$$P = \frac{mV_0 b}{\hbar a} \quad \text{with} \quad \alpha^2 = \frac{2mE}{\hbar^2}$$

or
$$E = \frac{\alpha^2 \hbar^2}{2m} \quad \text{with} \quad k = \frac{2\pi}{\lambda}$$

Equation (1) implies that an electron moving in a periodic potential lattice can have energy values only between allowed region for which left hand side lies between $+1$ and -1 . With the help of equation (1) it is possible to plot total energy (E) of electron versus wave number.

The curve is shown in fig. The right hand side of equation (1) becomes ± 1 for values of k given by

$$k = \frac{n\pi}{a}, \quad n = \pm 1, \pm 2, \dots$$

Hence the discontinuity of curve occurs for $k = \frac{n\pi}{a}$

The dotted curve shows the free electron parabola

$$E = \frac{\hbar^2 k^2}{2m}$$

From the graph it is clear that the electron has allowed energy values in the region or
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zone extending from $k = -\frac{\pi}{a}$ to $k = +\frac{\pi}{a}$. This zone is called the first **Brillouin zone**.

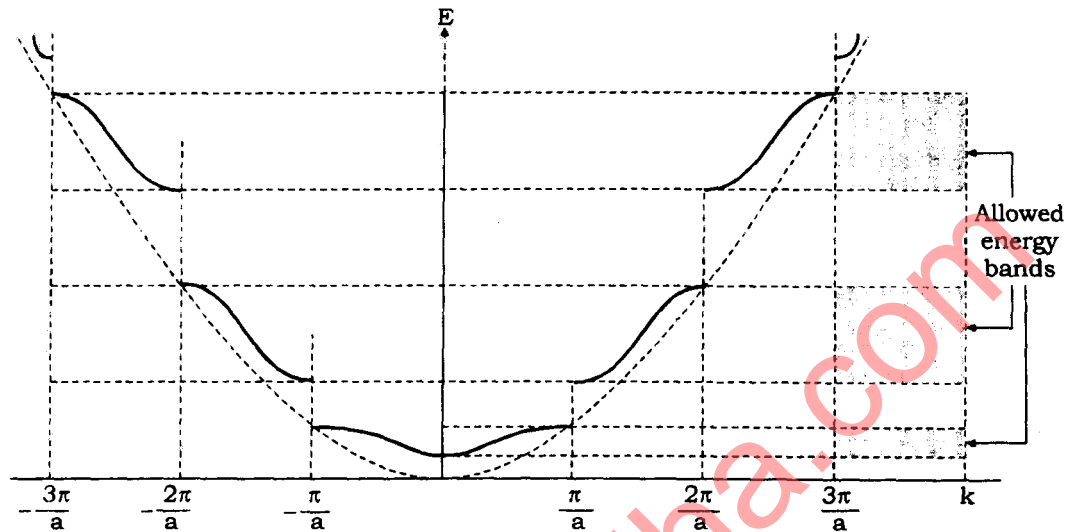
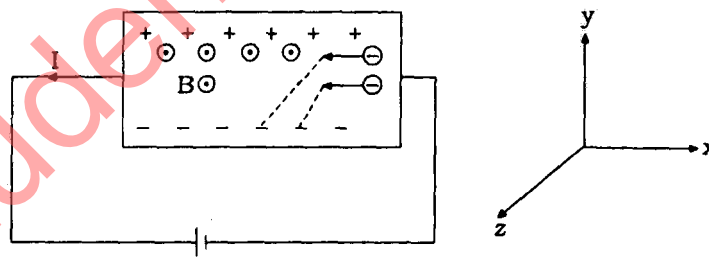


Fig. Plot energy vs wave number for Kronig Penney Model

Ans. 5. (b) Hall Effect

Definition : If a current carrying conductor (or semiconductor) is placed in a transverse magnetic field a voltage is produced perpendicular to both the direction of current and the direction of magnetic field. The voltage so developed is called Hall voltage and the phenomenon is called the hall effect.

Expression : The Hall voltage depends on the strength of the current and the magnetic field applied. The Hall effect may be explained by the motion of charged particles in electric and magnetic fields.



Consider a conductor having rectangular cross-section and carrying a current (I) along X-axis.

A uniform magnetic field B is applied along Z-axis, it is found that a voltage is developed along Y-axis in a direction both perpendicular to current (I) and magnetic field (B). The voltage applied along X-axis causes electric field along positive X-axis, which causes conduction electrons to accelerate along negative X-axis and hence they gain a drift velocity (v_d) along negative X-axis. In the presence of magnetic field, a magnetic Lorentz force acts on the electrons given by

$$\vec{F}_m = q(\vec{v}_d \times \vec{B})$$

Hence

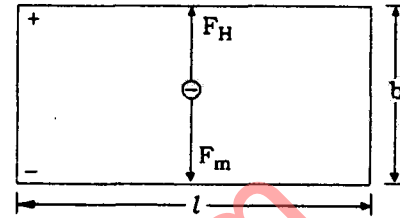
$$q = -e, \vec{v}_d = v_d \hat{i}, \vec{B} = B \hat{k}$$

\therefore

$$\vec{F}_m = -e(-v_d \hat{i}) \times B \hat{k} = -ev_d B \hat{j}$$

Clearly the magnetic force cause the electron to accelerate along negative Y-axis.

These electrons are accumulated along negative Y-axis; thus a potential difference is induced along Y-axis, this potential difference cause an electric force on electrons which opposes the Lorentz magnetic force. The accumulation process continues until the electron force (called the Hall force) balance the Lorentz magnetic force, i.e.,



$$F_H = F_m \quad \dots(1)$$

If E_H is Hall electric field, then

$$F_H = eE_H \quad \dots(2)$$

\therefore Equation (1) gives

$$eE_H = ev_d B \quad \text{(in magnitudes)}$$

\Rightarrow

$$E_H = v_d B \quad \dots(3)$$

The current density J_x is given by

$$J_x = -nev_d \quad \dots(4)$$

where n is number density of electrons in m^{-3} .

Dividing (3) and (4), we get

$$\frac{E_H}{J_x} = -\frac{B}{ne} \Rightarrow E_H = -\frac{B}{ne} J_x \quad \dots(5)$$

Clearly

$$E_H \propto J_x B$$

the coefficient of proportionality is called the Hall coefficient denoted by R_H . Thus

$$\text{Hall coefficient } R_H = \frac{E_H}{J_x B} = -\frac{1}{ne} \quad \dots(6)$$

Thus the Hall coefficient is defined as the ratio of Hall electric field per unit current density and the transverse magnetic field.

Prove $\phi = \mu H$: The mobility of a charge carrier (μ) is defined as the drift velocity per unit electric field, i.e.,

$$\mu = \frac{v_d}{E_x} \Rightarrow v_d = \mu E_x \quad \dots(7)$$

Substituting this value in (3), we get

$$E_H = \mu E_x B \quad \dots(8)$$

From equation (6)

$$E_H = J_x B R_H \quad \dots(9)$$

Comparing (8) and (9), we get

$$J_x B R_H = \mu E_x B \Rightarrow \mu = \frac{R_H J_x}{E_x} \quad \dots(10)$$

But $\frac{J_x}{E} = \sigma$, electrical conductivity

$$\mu = \sigma R_H \quad \dots(11)$$

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From equation (8)
$$\mu = \frac{E_H}{E_x} B = \tan \phi B \quad \dots(12)$$

where $\tan \phi = \frac{E_H}{E_x}$, the angle ϕ is called the Hall angle and B is the applied magnetic field.

Ans. 6. (a) Langevin's Theory of paramagnetism :

The theoretical explanation for paramagnetism was given by Langevin using the classical statistics of Boltzmann. He assumed that each atom consists of a permanent magnetic moment m , and that the only force acting on the atoms is that due to the external field B . If a given atomic or molecular dipole is pointing in a direction making an angle θ with B ; its magnetic potential energy is, $W = -mB \cos \theta$. According to Boltzmann's classical statistics the number of atoms or molecules per unit volume having an energy in the range W and $W + dW$ is,

$$dn = Ce^{-W/kT} dW \quad \dots(1)$$

where C is a constant to be determined, k is Boltzmann constant and T is the absolute temperature.

On differentiating the potential energy expression, we get

$$dW = mB \sin \theta d\theta \quad \dots(2)$$

Substituting the values of W and dW in equation (1), we get

$$dn = Ce^{mB \cos \theta / kT} \cdot mB \sin \theta d\theta \quad \dots(3)$$

Now we shall integrate this expression over all possible orientations, i.e., for θ from 0 to π , in order to find the total number of atoms or molecules n per unit volume of the substance. Thus,

$$n = \int dn = \int_0^\pi Ce^{mB \cos \theta / kT} \cdot mB \sin \theta d\theta$$

$$C = \frac{n}{\int_0^\pi Ce^{mB \cos \theta / kT} \cdot mB \sin \theta d\theta} \quad \dots(4)$$

This gives,

The component of each dipole moment parallel to B is $m \cos \theta$, and so the resultant magnetic moment due to all the dn atoms or molecules along the field is, $dn m \cos \theta$. The intensity of magnetization I which is the magnetic moment per unit volume is thus given by

$$\begin{aligned} I &= \int_0^\pi m \cos \theta \cdot dn \\ &= C \int_0^\pi m^2 Be^{mB \cos \theta / kT} \cdot \sin \theta \cos \theta \cdot d\theta \quad [\text{using eq. (3)}] \\ &= nm^2 B \frac{\int_0^\pi e^{mB \cos \theta / kT} \cdot \sin \theta \cos \theta \cdot d\theta}{\int_0^\pi e^{mB \cos \theta / kT} \cdot \sin \theta \cos \theta \cdot d\theta} \quad [\text{using equation (4)}] \\ &= nm \frac{\int_0^\pi e^{mB \cos \theta / kT} \cdot \sin \theta \cos \theta \cdot d\theta}{\int_0^\pi e^{mB \cos \theta / kT} \cdot \sin \theta \cos \theta \cdot d\theta} \quad \dots(5) \end{aligned}$$

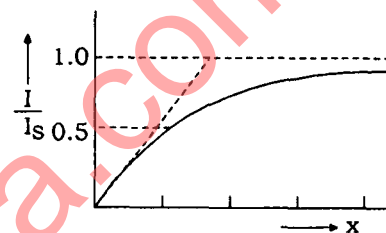
Let us write $mB/kT = x$ and $\cos \theta = y$, then equation (5) becomes,

$$I = nm \frac{\int_{-1}^{+1} e^{xy} y dy}{\int_{-1}^{+1} e^{xy} dy} = nm \frac{\left[\frac{ye^{xy}}{x} - \frac{e^{xy}}{x^2} \right]_{-1}^{+1}}{\left[\frac{e^{xy}}{x} \right]_{-1}^{+1}}$$

$$= nm \left[\frac{(e^x + e^{-x})}{(e^x - e^{-x})} - \frac{1}{x} \right] = I_s \left[\coth x - \frac{1}{x} \right] \quad \dots(6)$$

where $I_s = nm$ represents the saturation value of I , i.e., when all the atoms or molecules turn parallel to the field.

A plot of the function $I/I_s = \coth x - 1/x$ (which is known as Langevin function) is shown in fig. For large values of x , the function tends to unity, saturation being reached when all the atomic or molecular dipoles are parallel to B . For small values of x the curve is linear and coincides with tangent to the curve at the origin. For ordinary magnetic field strengths x is small and we can expand $\coth x$ in equation (5) in a power series of x retaining only the important terms. Equation (6) then becomes,



$$I = I_s \left[\left(\frac{1}{x} + \frac{x}{3} \right) - \frac{1}{x} \right] = I_s \frac{x}{3}$$

$$= \frac{n \cdot m^2 B}{3kT} \quad \left[\begin{array}{l} \text{since } x = \frac{mB}{kT} \\ \text{and } I_s = mn \end{array} \right]$$

or

$$I = \frac{n \cdot m^2}{3kT} B = \frac{nm^2 \mu_0 H}{3kT} \quad \dots(7)$$

The volume susceptibility for a paramagnetic substance is thus given by

$$\chi_m = \frac{I}{H} = \frac{\mu_0 nm^2}{3kT} = \frac{\mu_0 I_s^2}{3nkT} \quad \dots(8)$$

Thus is same as Curie law

$$\chi_m = \frac{\lambda}{T}$$

where Curie constant

$$\lambda = \frac{\mu_0 I_s^2}{3nk} \quad \dots(9)$$

Weiss-Modification in Langevin theory of Paramagnetism

Weiss in 1907, modified Langevin theory of paramagnetism by introducing a new concept of internal molecular field.

Weiss assumed that in a paramagnetic substance there must exist a molecular field produced at any point by all the neighbouring molecules which is proportional to the magnetisation vector I , i.e.,

$$\text{Molecular field } H' = \lambda I$$

Thus the actual field acting upon a dipole is the sum of the applied field H and that arising from the presence of neighbouring dipoles, i.e., effective magnetising field may be

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expressed as

$$H_i = H + \alpha I \quad \dots(10)$$

assuming that the contributions of the neighbouring dipoles to the effective field is proportional to the magnetic polarisation. This form of dependence is not self evident but was chosen by Weiss because it leads to the desired form. The factor of proportionality α is known as the *molecular field coefficient*.

Substituting (10) in (7), we get

$$I = \frac{\mu_0 n m^2 (H + \alpha I)}{3kT} = \frac{\mu_0 I_s^2 (H + \alpha I)}{3n k T} \quad (\text{since } I_s = nm) \quad \dots(11)$$

$$\therefore \text{Volume susceptibility } \chi_m = \frac{1}{H} = \frac{\mu_0 n m^2 (H + \alpha I)}{3kTH}$$

$$\text{i.e.,} \quad \chi_m = \frac{\mu_0 n m^2}{3kT} + \frac{\alpha \mu_0 n m^2}{3kT} \chi_m$$

$$\chi_m \left(1 - \frac{\alpha \mu_0 n m^2}{3kT} \right) = \frac{\mu_0 n m^2}{3kT}$$

$$\therefore \quad \chi_m = \frac{\frac{\mu_0 n m^2}{3kT}}{1 - \frac{\alpha \mu_0 n m^2}{3kT}} = \frac{(\mu_0 n m^2 / 3k)}{\left(T - \frac{\alpha \mu_0 n m^2}{3k} \right)} = \frac{\lambda}{T - \Theta} \quad \dots(12)$$

Equation (12) represents required Curie Weiss law. The constant Θ is known as Curie temperature.

Ans. 6. (b) Magnetic Susceptibility :

In simple isotropic materials the intensity of magnetisation I is found to be proportional to magnetising field H and parallel to it.

$$I \propto H$$

or

$$I = \chi_m H$$

Where χ_m is dimensionless constant and is called the magnetic susceptibility and is a characteristic of the medium, i.e., the magnetic susceptibility is defined as the ratio of intensity of magnetisation (I) to the magnetising field (H).

Curie - Weiss Law : The susceptibility of a paramagnetic substance decreases with a rise of temperature and is inversely proportional to temperature.

$$\chi \propto \frac{1}{T}$$

Ans. 7. (a) The London Equations :

To explain the Meissner effect in superconductors $F. London$ and $H. London$ in 1935 proposed a modification of the electrodynamics-equations. Two new equations have come out of their work which, besides explaining the Meissner effect, also explain the observations on thin films. These equations are known as the London equations. We now derive these equations and see how far they are successful in accounting for observations on the superconducting state

The assumption of zero resistivity leads to the acceleration equation

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$$m \frac{d\mathbf{v}}{dt} = -e\mathbf{E} \quad \dots(1)$$

as $\mathbf{J} = -nev$ where n is number of electrons per unit volume,

$$\begin{aligned} \therefore \frac{d\mathbf{J}}{dt} &= -ne \frac{d\mathbf{v}}{dt} \\ &= -ne \cdot \left(\frac{-e\mathbf{E}}{m} \right) \quad [\text{using (1)}] \end{aligned}$$

$$\text{or} \quad \frac{d\mathbf{J}}{dt} = \frac{ne^2}{m} \mathbf{E} \quad \dots(2)$$

It must be mentioned that here only the superconducting electrons are under consideration, and not all the electrons : as superconductor can be supposed as composed of both 'normal' and superconducting' electrons. The normal electrons behave like electrons in a non-conductor, and thus are of no interest to use. Further, the conducting electrons are being assumed to respond to electric field just as free electrons do.

Maxwell's equation of curl \mathbf{E} is

$$\text{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \dots(3)$$

$$\text{curl} \mathbf{H} = \mathbf{J} \quad \dots(4)$$

Now, taking the curl of both sides of (2), we have with

$$\text{curl} \frac{d\mathbf{J}}{dt} = \frac{ne^2}{m} \text{curl} \mathbf{E}$$

$$\text{Using (3),} \quad \text{curl} \frac{d\mathbf{J}}{dt} = -\frac{ne^2}{m} \frac{\partial \mathbf{B}}{\partial t}$$

$$\text{or} \quad \text{curl} \left(\Lambda \frac{d\mathbf{J}}{dt} \right) = -\mathbf{B} = -\mu_0 \mathbf{H}$$

$$\text{where} \quad \Lambda = \frac{m}{ne^2} \quad \dots(5)$$

This is rewritten as from (4), we have $\frac{d\mathbf{J}}{dt} = \text{curl}(\mathbf{H})$.

\therefore Equation (5) gives

$$\text{curl} \left(\Lambda \frac{d\mathbf{J}}{dt} \right) = \text{curl} \text{curl}(\Lambda \mathbf{H}) = -\mu_0 \mathbf{H} \quad \dots(6)$$

$$\text{As} \quad \text{curl} \text{curl}(\Lambda \mathbf{H}) = \text{grad} \text{div}(\Lambda \mathbf{H}) - \nabla^2(\Lambda \mathbf{H}) \quad \text{and} \quad \text{div} \mathbf{H} = 0$$

$$\therefore \text{Equation (6) gives} \quad \nabla^2(\Lambda \mathbf{H}) = \mu_0 \mathbf{H} \quad \dots(7)$$

Integration of (7) with respect to time gives

$$\Lambda \nabla^2(\mathbf{H} - \mathbf{H}_0) = \mu_0(\mathbf{H} - \mathbf{H}_0) \quad \dots(8)$$

where \mathbf{H}_0 is the constant of integration, and denotes the field at time $t = 0$. Now that equation (8) is a direct consequence of the Maxwell equations and the acceleration equation. Further, the currents are considered as the only internal source of magnetic field in this

discussion; no magnetisation as such has been introduced, and \mathbf{B} is taken simply equal to $\mu_0 \mathbf{H}$.

Let us now pay attention to the fact that the equation (8) admits the particular solution tells us that we cannot have frozen-in fields. We conclude that (8) has more general solution than nature allows in superconductors, and thus needs a slight modification before it can be of considerable use in the description of behaviour of superconductors.

According to F. London and H. London, we should eliminate \mathbf{H}_0 . This we do by abandoning the acceleration equation, and taking instead

$$\text{curl} \left(\Lambda \frac{m \mathbf{J}}{ne^2} \right) = -\mu_0 \mathbf{H} \quad \dots(9)$$

as the fundamental equation in a superconductor.

Proceeding again as before, we now obtain

$$\frac{m}{ne^2} \nabla^2 \mathbf{H} = \mu_0 \mathbf{H} \quad \dots(10)$$

Note that this does not admit a constant field as a solution. Thus (9) is the correct starting equation and not (5). A slightly different form of (9) is called the London equation. This form is obtained by introducing the vector potential \mathbf{A} by the definition $\text{curl } \mathbf{A} = \mu_0 \mathbf{H}$:

$$\mathbf{J} = -\frac{ne^2}{m} \mathbf{A} \quad \dots(11)$$

This is the London equation.

Explanation of Meissner Effect: Let us now see how the London equations lead to the Meissner effect and the flux penetration through thin films of superconductors. In the treatment we shall assume that we neglect the effects of the normal current (which is a valid assumption only at low enough temperatures) and also that we are dealing with frequencies which are low enough that we can neglect the effects of displacement current. By one of Maxwell equations, we can then write

$$\text{curl } \mathbf{H} = \mathbf{J} \quad \dots(12)$$

After taking curl, we obtain

$$\text{curl curl } \mathbf{H} = \text{curl } \mathbf{J}$$

$$\text{grad div } \mathbf{H} - \nabla^2 \mathbf{H} = \text{curl } \mathbf{J}$$

as

$$\text{div } \mathbf{H} = 0 \quad \dots(13)$$

$$-\nabla^2 \mathbf{H} = \text{curl } \mathbf{J}$$

Then, using the London equation (11), we obtain

$$\nabla^2 \mathbf{H} = \frac{ne^2}{m} \text{curl } \mathbf{A} = \frac{ne^2}{m} (\mu_0 \mathbf{H}) \quad \dots(14)$$

Or, putting

$$\lambda = \left(\frac{m}{\mu_0 ne^2} \right)^{1/2}$$

$$\nabla^2 \mathbf{H} = \frac{1}{\lambda^2} \mathbf{H} \quad \dots(15)$$

Equation (15) is a simple differential equation which has the solution.

$$H = H_0 e^{-x/\lambda} \quad \dots(16)$$

where x is the distance from the surface measured into the specimen and H_0 is the field at the surface. We may note that the solution (16) agrees with the experimental observation

that the field does not drop to zero abruptly at the surface, but runs into a certain region below the surface over which it gradually decays from the constant value H_0 at the surface *i.e.*, field penetrates in the specimen. The thickness of the penetrated region is measured by δ ; a typical experimental value of δ in a metal at OK is 500 Å. The Meissner effect is also accounted for as the specimens having dimensions much greater than δ can be considered, with good approximation, as if they have $\delta = 0$, *i.e.*, the ones in which the field has not penetrated at all.

Ans. 7. (b) Applications of super-conductors :

(i) Superconducting electromagnets that carry large resistanceless current and therefore produce large magnetic field ($\approx 5-10$ Tesla) can be constructed if we use superconducting wire for magnet windings, currents of the order 100 amperes can be carried by very fine wires. Thus small-size magnets can be constructed.

(ii) Superconducting cables can be used to transmit electric power over long distances without resistive losses. This would be economical, however, if the cost of keeping the cable below its critical temperature T_c is less than the value of the power that would be lost using ordinary cables. Materials are now available having $T_c = 20$ K.

(iii) Because superconductors are diamagnetic, they can be used to shield out unwanted magnetic flux, as in shaping the magnetic lens system of an electron microscope.

8. (a) Classical Theory of Ferromagnetism

Introducing the idea of domains Weiss exactly made the same assumptions for classical theory for ferromagnetism as for paramagnetics given by relation.

$$H_i = H + \alpha I \quad \dots(1)$$

When the molecular field coefficient α is positive, the possibility of spontaneous magnetisation of these domains arises even in the absence of external magnetic field. The value of such a spontaneous magnetisation due to internal molecular field may be found by putting the external field H to zero in the above relation. This relation can also determine the condition for spontaneous magnetisation.

Since the domains are assumed to obey the general theory of paramagnetism,

$$\frac{I}{I_s} = \coth x - \frac{1}{x} \quad \dots(2)$$

where

$$x = \frac{mB_i}{kT} = \frac{\mu_0 m H_i}{kT} = \frac{\mu_0 m (H + \alpha I)}{kT} \quad \dots(3)$$

Since the dipole aligning force $H_i = (H + \alpha I)$ depends partially upon the degree of alignment (magnetic polarisation) already prevailing, any increase in magnetic polarisation will result in an increase in the aligning force, which in turn, will result in an increase in the magnetic polarisation (or magnetisation) etc. In short a cumulative effect is to be expected. In this way magnetisation once induced, could occur without the application of an external field. Let us now investigate the conditions under which such spontaneous magnetisation could take place. When the applied field is $H = 0$, equation (3) reduces to

$$x = \frac{\mu_0 m \alpha I}{kT} = \alpha \frac{\mu_0 I_s}{nkT} \quad \dots(4)$$

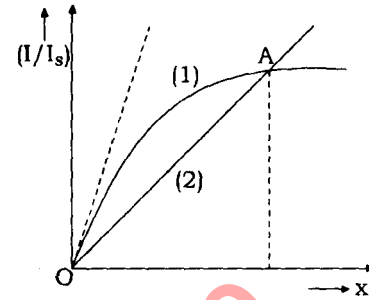
Since

$$I_s = nm$$

Equation (4) may be expressed as

$$\frac{I}{I_s} = \frac{nkT}{\alpha \mu_0 I_s^2} x \quad \dots(5)$$

Equations (2) and (5) constitute a pair of equations which may be solved simultaneously for two variables I/I_s and x . The solutions are most easily obtained by graphical method, as shown in the fig. represent the curves corresponding to two equations (2) and (5); equation (2) gives the Langevin's curve 1 while equation (5) gives a straight line 2 passing through the origin whose slope equal to



$\frac{nkT}{\alpha\mu_0 I_s^2}$ increases with T .

The curves (1) and (2) intersect at origin O and at another point A, so that the solutions are

$$\frac{I}{I_s} = 0 \quad \text{or} \quad AP.$$

But $I/I_s = 0$ cannot be a true solution, since there is spontaneous magnetisation by supposition. Hence the only correct solution $I/I_s = AP$, i.e., a non-zero point of intersection of two curves.

As the slope of the straight line curve (2) increases with T , if this slope coincides or becomes greater than that of tangent at the origin to the Langevin curve, then there will not exist a non-zero point of intersection of two curves; hence the spontaneous magnetisation will not occur. On the other hand, we know that the slope of the tangent at the origin to the Langevin curve is $1/3$, since $I/I_s = x/3$, when x is small.

From these considerations, the condition of stable spontaneous magnetisation may be expressed as

$$\frac{nkT}{\alpha\mu_0 I_s^2} < \frac{1}{3}$$

i.e.,

$$T < \frac{\alpha\mu_0 I_s^2}{3nk}$$

But $\alpha\mu_0 I_s^2 / 3nk = \Theta$, the Curie point; hence $T = \Theta$.

Hence below Curie point Θ a state of magnetisation is possible without an applied field. The degree of spontaneous magnetisation depends on the temperature approaching the saturation value as the temperature approaches absolute zero, since the non-zero point of intersection will take place at

$$I/I_s = 1 \quad \text{for} \quad x = \mu_0 mH/kT \rightarrow \text{infinite when } T = 0.$$

It may be noted from the relation $\Theta = \alpha\mu_0 I_s^2 / 3nk$ that the absolute value of Θ is greater if the magnetic moment of domains (I_s) and molecular field coefficient α have large values. If Θ is greater, greater will be the transition temperature. Incidentally the Curie temperature for ferromagnetic elements Fe, Co and Ni are 770°C , 1131°C respectively. Above the Curie point Θ the spontaneous magnetisation no longer occurs and ferromagnetic properties disappear and the substance becomes paramagnetic. At temperatures, not too near the transition point the Curie-Weiss law is obeyed as already shown in preceding section.

$$\chi_m = \frac{\lambda}{(T - \Theta)}$$

The chief drawbacks of Weiss theory are that (i) it could not explain why and how internal fields between the molecules of ferromagnetic materials possess such large values, and (ii) why the linear relationship expressed by Curie-Weiss law breaks near Curie point.

Ans. 8. (b) Effect of illumination on photoconductivity with and without concept of trap:

We consider a crystal with N electron traps levels per unit volume. Supposing then that the temperature is sufficiently low in relation to the relevant ionization energies so that the

concentration of thermal carriers may be neglected, we modify $\left(\frac{dx}{dt} = L - AnP = L - An^2\right)$ as

$$\frac{dn}{dt} = L - An(n + N) \quad \dots(1)$$

Where n is the electron concentration in the conduction band. For simplicity, we have assumed that the proportionality coefficient A is the same for electron-hole recombination as for electron traps capture, and have omitted the effect of thermal ionization of carriers from traps back into the conduction band; this effect would have added a term B_n to the right hand side of eqn. (1). In the steady state, we now therefore have

$$n_0 = (n_0 + N) = L / A \quad \dots(2)$$

Now it is usually difficult to grow crystals with trap concentrations N much less than 10^{14}cm^{-3} . Accordingly, we may discuss two limiting cases, one in which the carrier concentration n_0 may be very much less than this, perhaps only 10^8 or 10^{10}cm^{-3} , and another in which it is very much greater. The former case holds at low levels of illumination, and the latter at high levels of illumination. Thus, in the limit $n_0 \ll N$, we have

$$n_0 = L / AN \quad \dots(3)$$

in eq. $n_0 = \left(\frac{L}{A}\right)^{1/2}$. The photocurrent is now directly proportional to the illumination level L , which is in fairly satisfactory agreement. In the limit $n_0 > N$, the response is given by

$$n_0 = (L / A)^{1/2}, \quad \dots(4)$$

just as found earlier in the absence of traps. The experimental results act to confirm this change in the response with illumination level.

The decay of the carriers on switching off the light is given by

$$\log \frac{n + N}{n} - \log \frac{n_0 + N}{n_0} = NAt \quad \dots(5)$$

obtained by solving (1) with $L = 0$. In the limit $N > n_0$, it reduced to $n = n_0 \exp(-NAt)$, and so the time for the photocurrent to fall to e^{-1} of its initial value is

$$t_0 = 1 / NA \quad \dots(6)$$

Just compare this result with the earlier result eq. $t_0 = \frac{1}{An_0} = (LA)^{-1/2} = \frac{n_0}{L}$ in the

absence of traps - the presence of traps reduces the response time. Also, note the reduction in photoconductivity from (4).

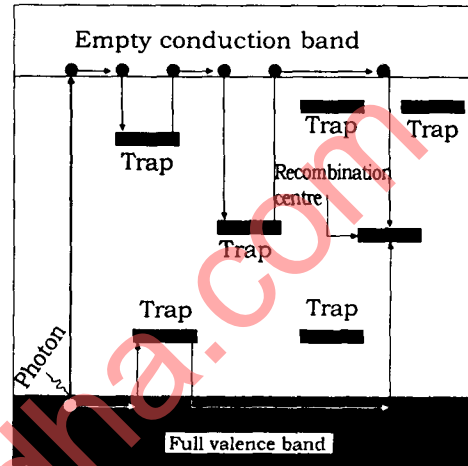


Fig. Influence of Traps in electron-hole recombination